THE CBOE SKEW INDEX®SM - SKEW®SM

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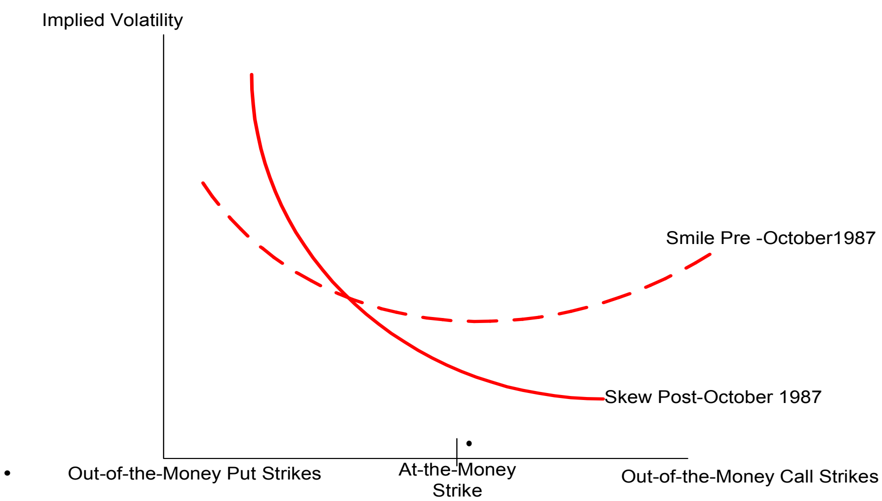
Introduction

Since it emerged from a smile in the wake of the crash of October 1987, the curve of S&P 500® implied volatilities, a.k.a. the smile or "skew", has been one the most-studied features of S&P 500 option prices. As illustrated in Chart 1, the smile has lost its symmetry and it is biased towards the put side.

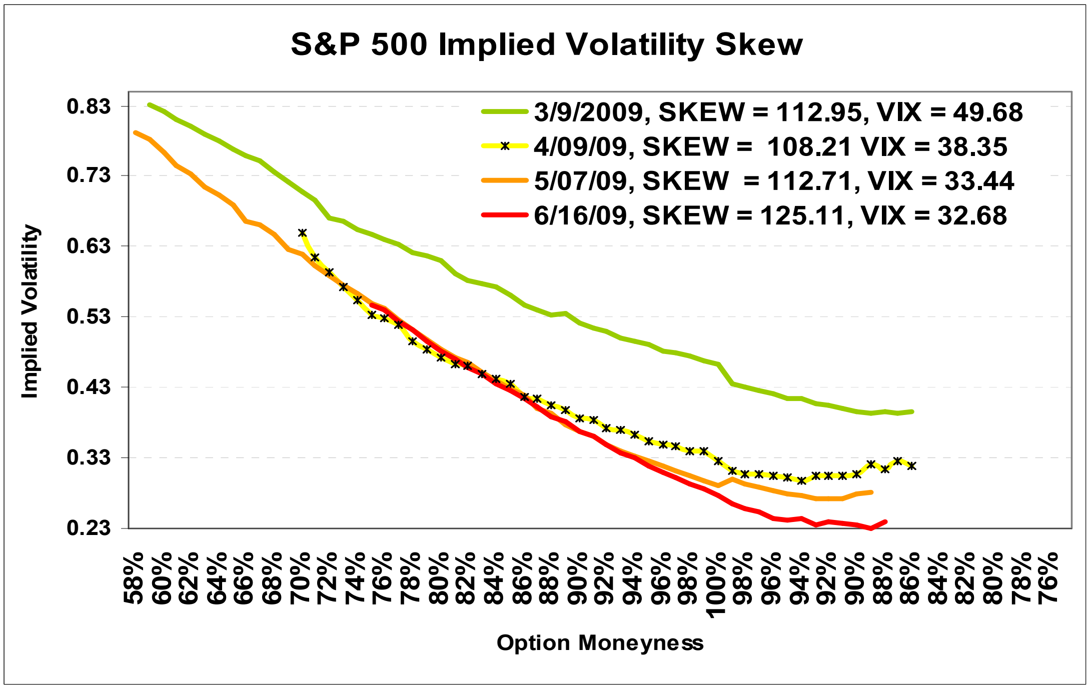
Chart 1. The S&P 500 Implied Volatility Curve Pre-and Post- 1987



Source: CBOE

To get at the core of the skew, the Chicago Board Exchange® (CBOE®) is introducing a new benchmark, the CBOE Skew Index® (SKEW). SKEW is a global, strike-independent measure of the slope of the implied volatility curve that increases as this curve tends to steepen. This is illustrated in Figure 2 with snapshots of the S&P 500 implied volatility curve, SKEW and the CBOE Volatility Index® (VIX®) from March 2009 to June 2009. There is no significant change in SKEW or the overall slope of the implied volatility curve between March and May 2009. By mid-June, SKEW is significantly higher, and the implied volatility curve is noticeably steeper. Chart 2 also illustrates the low correlation between variations in SKEW and VIX. SKEW is calculated from S&P 500 option prices using a method similar to that used for VIX.

Chart 2. Snapshots of Curve of S&P 500 Implied Volatilities, May to June 2009



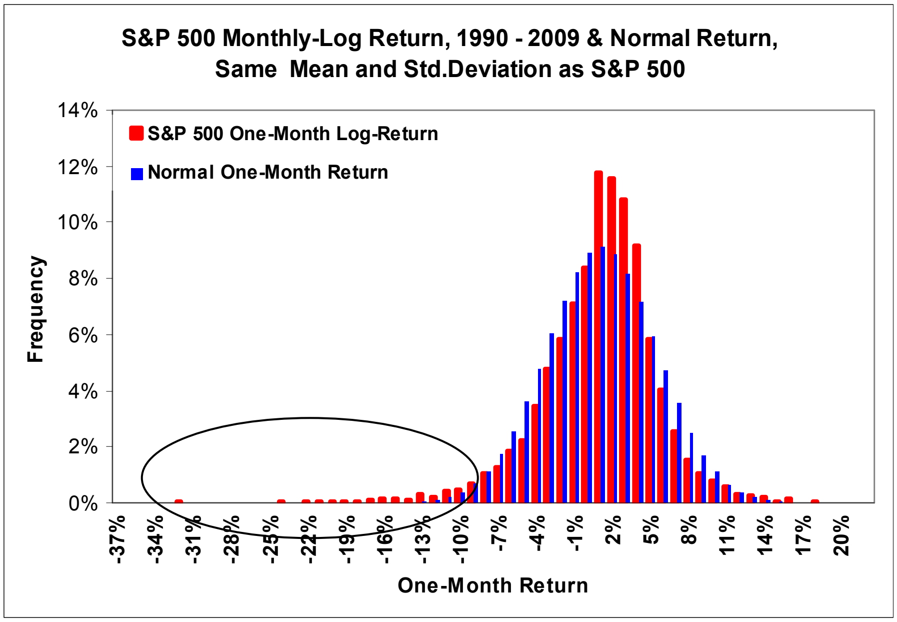
Scaled Differences of Implied Volatilities (IV), SKEW and VIX				
	9-Mar-09	9-Apr-09	7-May-09	16-Jun-09
(90% IV -110% IV) / 100% IV	0.27	0.25	0.30	0.48
SKEW	112.95	108.21	112.71	125.11
VIX	49.68	38.35	33.44	32.68

Source: CBOE

To understand SKEW, it helps to recall why the curve of S&P 500 implied volatilities no longer smiles. The change reflects the fact that investors now prize low strike puts more than they do high strike calls. Why? Because the October 1987 crash has sensitized the market to the possibility of large downwards jumps in the S&P 500. The distribution of S&P 500 log-returns ("S&P 500 distribution") is unlikely to be normal if there are large jumps in returns. Jumps fatten the weights of the tails and asymmetric jumps skew the distribution. The standard deviation of returns is then insufficient to characterize risk and the probability of returns two or three standard deviations below the mean is not negligible, as it is under a normal distribution.

Chart 3 confirms that the S&P 500 distribution is far from normal. It carries "tail risk": (a) the frequency of outlier returns is greater than for a normal distribution and (b) the distribution has a negative skew. This means that VIX, a proxy for the standard deviation of the S&P 500 distribution, may not fully capture the perceived risk of a cash or derivative investment in the S&P 500 or in correlated assets. In light of this, CBOE has developed a complementary indicator that measures perceived tail risk. That indicator is SKEW.

Chart 3. Frequency Distribution of S&P 500 Log-Return



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Source: CBOE

Similar to VIX, SKEW is calculated from the price of a tradable portfolio of out-of-the-money S&P 500 (SPX[SM]) options. This portfolio constitutes an exposure to the skewness of S&P 500 returns and its price encapsulates how the market prices tail risk. A detailed description of the SKEW methodology and the derivation of the SKEW portfolio are in Appendix I. A numerical example of the calculation of SKEW is in Appendix II.

1. Definition of SKEW

SKEW is derived from the price of S&P 500 skewness, denoted by S. S is defined similarly to a coefficient of statistical skewness:

$$S = E[(\frac{R-\mu}{\sigma})^3]$$

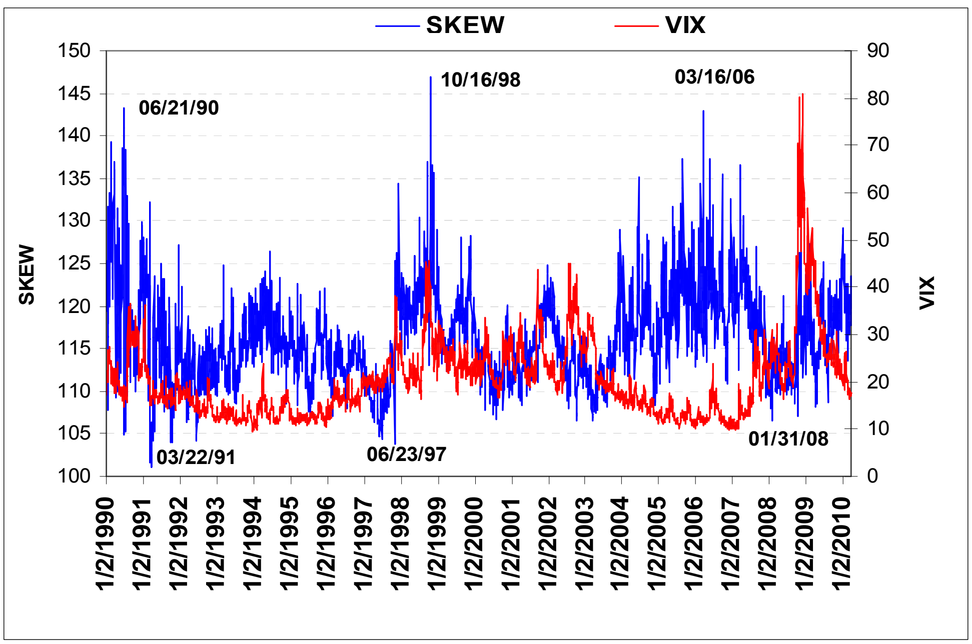
R is the 30-day log-return of the S&P 500, μ is its mean, and σ is its standard deviation; $x = (\frac{R-\mu}{\sigma})^3$ represents a skewness payoff, and S = E[x] is its market price, a risk adjusted expectation of x.

S is calculated from a portfolio of S&P 500 options that mimics an exposure to a skewness payoff. Since S tends to be negative and to vary within a narrow range (-4.69 to - .10 between 1990 and 2010), it is inconvenient to use it as an index. S is therefore transformed to SKEW by the following linear function:

$$SKEW = 100 - 10 * S$$

With this definition, SKEW increases as S becomes more negative and tail risk increases.

Chart 4. SKEW 1990 – 2010



Source: CBOE

2. Behavior of SKEW

a. Level of SKEW

Chart 4 follows the history of SKEW and VIX from 1990 to 2010. The minimum value of SKEW over this period is 101 and its maximum is 147. Table 1, a table of the historical frequencies of SKEW values provides further information about the range of SKEW. In this table, the frequency associated with a value of SKEW is the percentage of times that SKEW lied in the range between that value and the value above. For example, 17.55% of the time, SKEW ranged between 115 and 117.5.

Table 1. Historical Frequencies of SKEW

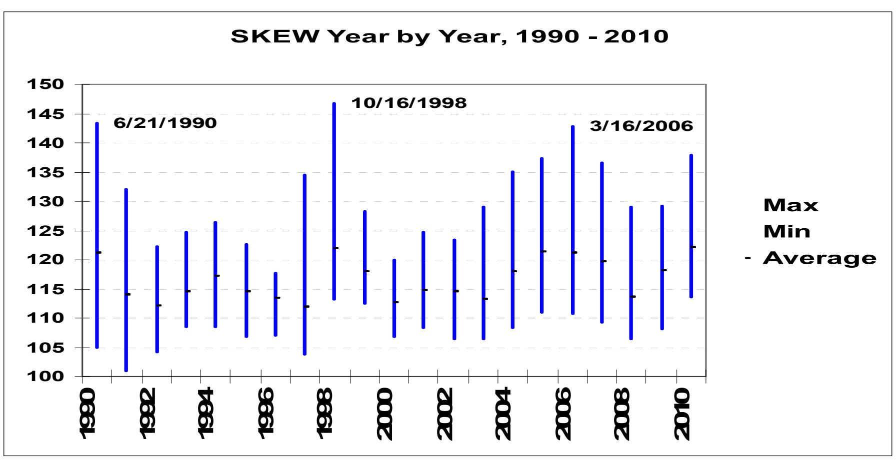
SKEW	Frequency 1990 - 2010	SKEW	Frequency 1990 - 2010
100.00	0.00%	127.50	3.07%
102.50	0.10%	130.00	1.55%
105.00	0.42%	132.50	0.79%
107.50	1.59%	135.00	0.40%
110.00	7.21%	137.50	0.30%
112.50	15.38%	140.00	0.10%
115.00	20.47%	142.50	0.02%
117.50	17.55%	145.00	0.04%
120.00	14.96%	147.50	0.02%
122.50	10.14%	150.00	0.00%
125.00	5.90%		

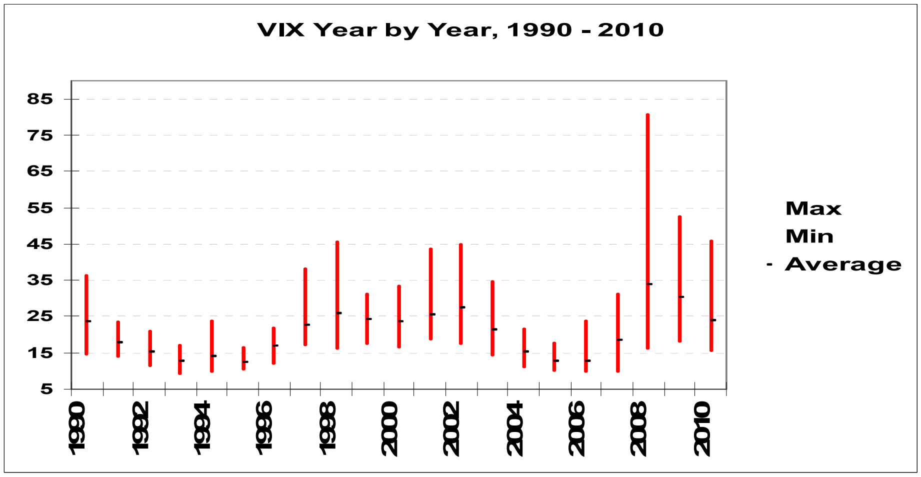
Source: CBOE

b. Annual Range of SKEW

Chart 5 shows the annual highs and lows of SKEW and VIX from 1990 to 2010.

Chart 5. Annual Highs and Lows of SKEW and VIX





Source: CBOE

c. Interpretation of SKEW

To get a sense of what high or very high tail risk means, one can translate the value of SKEW to a risk-adjusted probability that the one-month S&P 500 log-return falls two or three standard deviations below the mean, and use VIX as an indicator of the magnitude of the standard deviation. When SKEW is equal to 100, the distribution of S&P 500 log-returns is normal, and the probability of returns two standard devations below or above the mean is 4.6% (2.3% on each side); the probability decreases to .3% (.15% on each side) for three standard deviations. For a non-normal distribution, comparable probabilities are approximated[1] by adding a skewness term to the normal distribution. The resulting probabilities are shown in Table 2. The probability of a return two standard deviations below the mean gradually increases from 2.3% to 14.45% as SKEW increases from 100 to 145. The probability of a return three standard deviations below the mean increases from .15% to .45% as SKEW increases from 100 to 105 and increases to 2.81% when SKEW reaches 145.

[1] The probabilities shown are risk-adjusted estimates based on overlaying risk-neutral skewness over a normal distribution, as done in a Gram-Charlier expansion of the normal distribution, as in D. Backus, S. Fioresi and K. Li (1997). The risk-neutral kurtosis is omitted from the expansion because, as shown by G. Bakshi, N. Kapadia and D. Madan (2003), it is empirically not significant.

Table 2. Estimated Risk-Adjusted Probabilities of S&P 500 Log Returns Two and Three Standard Deviations below the Mean

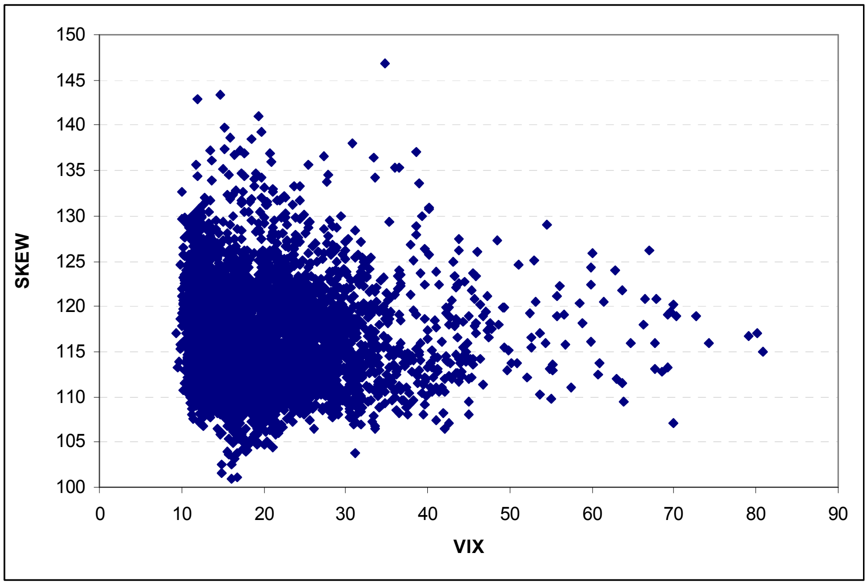
Estimated Risk Adjusted Probability		
SKEW	S&P 500 30-Day Log Return	
	2 Std. Dev	3 Std. Dev.
100	2.30%	0.15%
105	3.65%	0.45%
110	5.00%	0.74%
115	6.35%	1.04%
120	7.70%	1.33%
125	9.05%	1.63%
130	10.40%	1.92%
135	11.75%	2.22%
140	13.10%	2.51%
145	14.45%	2.81%

Source: CBOE

The probabilities in Table 2 apply to return variations measured in standard deviation units. VIX, a proxy for the standard deviation is therefore needed to complete the picture of risk. VIX captures the first layer of perceived risk, as it tells how far on average the S&P 500 log-return is likely to stray on either side of its mean, including the risky downside. Once this is gauged, SKEW catches the additional layer of risk implied by the left tail of the distribution.

Perceived tail risk increases when market participants increase their probability of a catastrophic market decline, what has come to be called a "black swan". As illustrated by Chart 6, a scatter plot of VIX and SKEW, high values of SKEW occur in conjunction with both low or high values of VIX. Note that the upper bound of SKEW values decreases as VIX rises to extreme values above 40. The probable reason is that VIX surges during periods of crashing stock prices, when a repeat crash may not be viewed as that likely.

Chart 6. Scatter plot of SKEW and VIX, 1990 – 2010



Source: CBOE

d. Volatility of SKEW

The definition of SKEW magnifies variations of S by 10, but it reduces its daily percentage variations as well as its realized volatility. To illustrate, the absolute daily change in S varies between .0001 and 3.40 around an average of .18 and that of SKEW varies between .001 and 34 around an average of 1.76 . The absolute daily rate of change of S varies between .004% and 375% around an average of 11.1%, that of SKEW varies between .001% and 25% around an average of 1.49%. The 30–day realized volatility of S is 872, and that of SKEW is 110. This compares to a 30-day realized volatility of 372 for VIX.

3. One Last Thing You Should Know About SKEW

CBOE calculates a daily term structure of SKEW. Historical prices for the SKEW Index and for this term structure can be found on the CBOE website at http://www.cboe.com/micro/IndexSites.aspx under CBOE SKEW Indexes.

APPENDIX I. DERIVATION OF SKEW & SKEW PORTFOLIO

Derivation of SKEW

SKEW is defined as SKEW = 100 – 10 * S, where $S = E[\frac{(R - \mu)^3}{\sigma^3}]$, R is the 30-day log-return of the S&P 500, μ = E[R] is its expected value, and σ = E[(R- μ)^2] ^{½} is its standard deviation, with all expectations E[x] taken under the risk-neutral density.

S is easily recognized as the risk-neutral version of a coefficient of statistical skewness. S is also the expectation, or market price of a skewness payoff[2] determined by the asymmetry of the S&P 500 log-return. When the S&P 500 log-return is symmetric, the payoff is equal to 0, and when the S&P 500 log-return is biased toward negative (positive) values, the payoff is negative (positive).

In general, 30-day options are not available and S is derived by inter or extrapolation from S_{near} and S_{next}, the prices of skewness at adjacent expirations:

$$S = w\, S_{near} + (1-w)\, S_{next}$$

where w = $(T_{next}- T_{30})/(T_{next}- T_{near})$, and T_{near}, T_{next} and T_{30} are the times to expiration of the near and next term options expressed in minutes, and T is the number of minutes in 30 days.

To streamline exposition, in what follows, S refers to either S_{near} or S_{next}.

S is expanded as the following function of the prices P_1, P_2, and P_3 of the power payoffs R, R^2, and R^3 :

$$S = \frac{E[R^3] - 3E[R]E[R^2] + 2E[R]^3}{(E[R^2] - E^2[R])^{3/2}} = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^{\ 2})^{3/2}}$$

Similar to realized variance, power payoffs can be replicated by delta-hedging portfolios of at- and out-of-the money options[3]. Hence, the calculation of P_1, P_2, and P_3, and from there SKEW, is analogous to the calculation of VIX. The selection of S&P 500 contract months and the screening of S&P 500 series are the same, as is the inter or extrapolation of SKEW from SKEW-like measures at option expiration dates adjacent to 30 calendar days Where the two calculations diverge is in the equations that are applied to the option prices to derive P_1 , P_2 and P_3:

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[2] See G. Bakshi, N. Kapadia and D. Madan, Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, The Review of Financial Studies, 16(1), 101-143, 2003.

[3] P. Carr and D. Madan, Towards a Theory of Volatility Trading, Volatility, Robert Jarrow, ed., Risk Publications, pp. 417-427, 2002 show that any continuous payoff can be expanded in terms of sums of option payoffs. This is sufficient to expand power returns as sum of option payoffs. For realized variance, one also needs to apply Ito's lemma.

(1) $P_1 = \mu = E[R_T] = e^{rT}(-\sum_i \frac{1}{K_i^2} Q_{K_i} \Delta_{K_i}) + \varepsilon_1$

(2) $P_2 = E[R_T^2] = e^{rT}(\sum_i \frac{2}{K_i^2}(1-\ln(\frac{K_i}{F_0}))Q_{K_i}\Delta_{K_i}) + \varepsilon_2$

(3) $P_3 = E[R_T^3] = e^{rT}(\sum_i \frac{3}{K_i^2}\{2\ln(\frac{K_i}{F_0}) - \ln^2(\frac{K_i}{F_0})\}Q_{K_i}\Delta_{K_i}) + \varepsilon_3$

where

F_0	Forward index level derived from index option prices
K_0	First listed strike below F_0
K_i	Strike price of ith out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i <$ K0; both put and call if Ki=K0.
ΔKi	Half the difference between the strike on either side of Ki: $\Delta K_i = \frac{1}{2}(K_{i+1} - K_{i-1})$ (Note: For the minimum (maximum) strike, ΔKi is simply the distance to the next strike above (below).
r	Risk-free interest rate to expiration
Q(Ki)	The midpoint of the bid-ask spread for each option with strike Ki.
T	The time to expiration expressed as a fraction of a year.
ε_j	Adjustment terms compensating for the difference between K_0 and F_0

(4) $\varepsilon_1 = -(1 + \ln(\frac{F_O}{K_O}) - \frac{F_O}{K_O}),$

(5) $\varepsilon_2 = 2\ln(\frac{K_O}{F_O})(\frac{F_O}{K_O} - 1) + \frac{1}{2}\ln^2(\frac{K_O}{F_O}),$

(6) $\varepsilon_3 = 3\ln^2(\frac{K_O}{F_O})(\frac{1}{3}\ln(\frac{K_O}{F_O}) - 1 + \frac{F_O}{K_O})$

SKEW Portfolio

Recall that SKEW = 100 – 10 S, where S prices a portfolio that replicates an exposure to 30 day-skewness, which we call the "skewness" portfolio. This implies that SKEW is also the price of a portfolio, namely the portfolio that overlays a position short 10 times the skewness portfolio over a money market position.

To determine the composition of the skewness portfolio, note that its payoff is a linear function of the power payoffs R, R^2 and R^3

$$\frac{(R \quad - \mu)^3}{\sigma^3} = \frac{R^3 - 3\mu R^2 + 3\mu^2 R - \mu^3}{\sigma^3} = a_1 R + a_2 R^2 + a_3 \ R^3 - (\frac{\mu}{\sigma})^3 .$$

where μ = E[R] = P_1, and σ = $E[(R- \mu)^2]^{½}$ = $E[R^2]$- $E[R]^2$= P_2 - P_1^2 .

Each power payoff is replicated by a portfolio of SPX options. Hence, the skewness portfolio is obtained by aggregating the three power portfolios and overlaying the combination on a money market position with payoff $-(\frac{\mu}{\sigma})^3$.

The compositions of the power portfolios are implicit in equations (1) to (3). Each consists of a strip of at- and out-of-the-money S&P 500 puts and calls weighted by different coefficients. The first strip is delta-hedged by a static position in S&P 500 forward contracts. The number of put or call options at strike K held in the three power strips are as follows:

R: $b_{1K} = -\frac{\Delta K}{K^2}$

R^2: $b_{2K} = 2\frac{\Delta K}{K^2} * (1 - \ln\frac{K}{F_0})$,

$R^{3:}$ $b_{3K} = 3\frac{\Delta K}{K^2} * (2\ln(\frac{K}{F_0}) - \ln^2(\frac{K}{F_0}))$

An exposure to the skewness payoff is constructed by aggregating the number of calls or puts at each strike.

The number of puts or calls held at strike K is equal to $\alpha_K = a_1 b_{1K} + a_2 b_{2K} + a_3 b_{3K}$. Afer substituting the expressions for a_i and b_{iK}, α_K is found equal to:

$$\alpha_K = \frac{3\Delta K}{\sigma^3 K^2}\{-\ln^2(\frac{K}{F_0}) + 2\ln(\frac{K}{F_0})(1+\mu)\} - \mu(\mu+2)\}$$

Substituting the different option and money market positions in SKEW, and taking into account the $100 multiplier of S&P 500 options, we finally obtain the SKEW portfolio:

Money market position : $e^{-rT}\{100 - 10\,[w*(-(\frac{\mu_{near}}{\sigma_{near}})^3) + (1-w) * (-(\frac{\mu_{next}}{\sigma_{next}})^3)]\}$

Portfolio of OTM options : - .01* 10 * w * $a_{K\ near}$ of near term option with strike $K_{,}$ and

- .01* 10 * (1-w)* $a_{K\ next}$ of next term option with strike $K_{.}$

The options at the near and next expiration are delta hedged by shorting 10 / M S&P 500 forward contracts, where M is the multiplier of an S&P 500 forward contract with the same expiration.

APPENDIX II THE SKEW CALCULATION STEP-BY-STEP

The calculation of SKEW proceeds in two parts. The first is to determine the composition of the portfolio of S&P 500 puts and calls to be used. This part proceeds exactly as for VIX. Once the portfolio is established, formulas (1) to (6) are applied to put and call prices to find S_{near} and S_{next}. This leads directly to SKEW. The determination of the SKEW portfolio is a by-product of the calculation.

The following hypothetical example of the calculation of SKEW on July 28, 2010, at 10:45 am Chicago time illustrates the calculation step-by step. An extract of the data used is shown in the example.

Step 1: ***Determinination of options in SKEW portfolio.***

- Selection of option months: The near and next-term options are usually the first and second SPX contract months. "Near-term" options must have at least one week to expiration; this requirement is intended to minimize pricing anomalies that might occur close to expiration. When the near-term options have less than a week to expiration, SKEW "rolls" to the second and third SPX contract months. For example, on the second Friday in June, SKEW would be calculated using SPX options expiring in June and July. On the following Monday, July would replace June as the "near-term" and August would replace July as the "next-term.".

  On July 28, 2010, August 2010 and September 2010 options are selected as the near and next term options.

- Calculation of time to expiration: The time to expiration of the selected options is calculated next. This is needed to calculated interest rate factors and also to interpolate between values at adjacent months to get the 30-day SKEW. The time to expiration is expressed as a fraction of a year, based on the number of minutes to expiration. The options are deemed to expire at 8:30 am Chicago time on the third Friday and a year is deemed to have 365 days.

August 2010 options expire on the 20th and September 2010 options expire on the 17th . At 10:45 am on July 28, 2010, the times to expiration of August and September 2010 options are equal to 0.065 and 0.142.

- Interest rate used: The second piece of data to calculate interest rate discount factors is the risk-free interest rate, r. r is the bond-equivalent yield of the U.S. T-bill maturing closest to the expiration dates of relevant SPX options. As such, the SKEW calculation may use different risk-free interest rates for near- and next-term options.

In this example, R = 0.00155 for both sets of options.

- Calculation of S&P 500 forward price and determination of at-the-money strike: Similar to VIX, is calculated from at and out-of-the-money puts and calls. The at-the-money strike is defined as the listed strike immediately below the S&P 500 forward price. To find the forward price, find the strike for which the difference between the midquotes of the call and put is at a minimum. Then calculate the forward price as
  $F = e^{rT} * (C - P) + K$, where T is the time to expiration, C and P are the call and put midquotes, and K is the strike at which minimum occurs.

As seen in Table 1 below, for August 2010 options, the strike at which the minimum of the midquote difference is attained is 1105. The 1105 row is highlighted in green. The forward price for August 2010 is equal to

$$F_{Aug} = e^{.00155 * 0.65} * (23.7 - 21.85) + 1105 = 1106.85.$$

The strike for September 2010 options also turns out to be 1105. By a similar calculation, F_{Sep} = 1106.45. Thus the at-the-money strike for both August and September is 1105. Puts at 1105 or below and calls at 1105 or above will be included in the calculation.

Table 1. Extract from August 2010 option data on July 28, 2010, 10:45 am Chicago time.

Strike	Put Bid	Put Ask	Put Midquote	delta k	Call Bid	Call Ask	Call Midquote	midcall - midput
690	0	0	**0**		414.4	418.5	**416.45**	
695	0	0	**0**		410	414.1	**412.05**	
700	0.05	0.1	**0.075**	5	405.3	410.3	**407.8**	407.725
705	0.05	0.1	**0.075**	5	401.6	405.7	**403.65**	403.575
710	0.05	0.1	**0.075**	5	396.3	400.3	**398.3**	398.225
715	0.05	0.1	**0.075**	5	389.6	393.7	**391.65**	391.575
720	0.05	0.1	**0.075**	5	385.3	390.3	**387.8**	387.725
...	...	...	...	...	...	...	...	...
1095	16.7	19.2	**17.95**	5	28.9	31.2	**30.05**	12.1
1100	19	20.5	**19.75**	5	25.7	27.6	**26.65**	6.9
1105	20	23.7	**21.85**	5	22.5	24.9	**23.7**	1.85
1110	23.5	25	**24.25**	5	20.6	22.1	**21.35**	-2.9
1115	25.1	28	**26.55**	5	18.5	20.8	**19.65**	-6.9
1120	28.3	30.1	**29.2**	5	14.3	17.8	**16.05**	-13.15
1125	30.9	32.9	**31.9**	5	13.7	14.9	**14.3**	-17.6
1130	34	36.1	**35.05**	5	11.3	12.6	**11.95**	-23.1
1135	36.6	38.8	**37.7**	5	9.2	10.7	**9.95**	-27.75
...	...	...	...	...	...	...	...	...
1205	97	100.2	**98.6**	5	0.05	0.9	**0.475**	-98.125
1210	100.2	104.1	**102.15**	5	0	0	**0**	-102.15
1215	106.9	110.1	**108.5**	5	0.05	1	**0.525**	-107.98
1220	111.9	115.1	**113.5**	5	0	0	**0**	-113.5
1225	116.8	120	**118.4**	5	0.1	0.3	**0.2**	-118.2
1230	121.8	125.3	**123.55**	5	0.05	0.3	**0.175**	-123.38
1235	125.3	129.3	**127.3**	5	0.05	0.25	**0.15**	-127.15
1240	130.7	134.7	**132.7**	5	0.05	0.7	**0.375**	-132.33
1245	136.7	140.7	**138.7**	5	0.05	0.2	**0.125**	-138.58
1250	140.7	144.7	**142.7**	5	0.05	0.1	**0.075**	-142.63
1255	146.7	150	**148.35**	5	0.05	0.3	**0.175**	-148.18
1260	151.7	155	**153.35**	5	0	0.9	**0**	
1265	156.7	160	**158.35**	5	0	0.25	**0**	

Source : CBOE

- <u>Elimination of strikes</u>. The SKEW calculation only uses at or out-of-the money strikes. In addition, only options with non-zero bid prices are used, and once two consecutive puts with 0 bid prices are found, all puts with lower strike prices are eliminated. Similarly, once two consecutive calls with 0 bid prices are found, all calls with higher strike prices are eliminated. In Table 1, all data for eliminated strikes are grayed out, and puts and calls eliminated because they were below and above two consecutive strikes with zero bids are not shown. Data for strikes ranging from 720 to 1095 and from 1135 to 1205 are replaced by dots (…) to provide a more compact display.

  For August 2010 options, this elimination process leaves puts with strikes from 700 to 1105 and calls with strikes from 1105 to 1255. Also leave out calls with strikes 1210 and 1220 because they have 0 bids. For September 2010 options, use puts with strikes from 725 to 1105 and calls with strikes from 1105 to 1280.

  One important note: as volatility rises and falls, the strike price range of options with non-zero bids tends to expand and contract. As a result, the number of options used in the SKEW calculation may vary from month-to-month, day-to-day and possibly, even minute-to-minute.

<u>Step 2</u> – *Calculate SKEW for both near-term and next-term options*

Recall that SKEW = 100 – 10 S, where S the price of skewness can be derived from the prices of S&P 500 options:

$$S = \frac{E[R^3] - 3E[R]E[R^2] + 2E[R]^3}{(E[R^2] - E^2[R])^{3/2}} = \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^{\ 2})^{3/2}}$$

(1) $P_1 = \mu = E[R_T] = e^{rT}\left(-\sum_i \frac{1}{K_i^{\ 2}} Q_{K_i} \Delta_{K_i}\right) + \varepsilon_1$

(2) $P_2 = E[R_T^2] = e^{rT}\left(\sum_i \frac{2}{K_i^{\ 2}}\left(1 - \ln\left(\frac{K_i}{F_0}\right)\right) Q_{K_i} \Delta_{K_i}\right) + \varepsilon_2$

(3) $P_3 = E\,[R_T^3] = e^{rT}\left(\sum_i \frac{3}{K_i^{\ 2}}\left\{2\ln\left(\frac{K_i}{F_0}\right) - \ln^2\left(\frac{K_i}{F_0}\right)\right\} Q_{K_i} \Delta_{K_i}\right) + \varepsilon_3$

$$\text{(4)}\quad \varepsilon_1 = -\left(1 + \ln\left(\frac{F_O}{K_O}\right) - \frac{F_O}{K_O}\right),$$

$$\text{(5)}\quad \varepsilon_2 = 2\ \ln\left(\frac{K_O}{F_O}\right)\left(\frac{F_O}{K_O} - 1\right) + \frac{1}{2}\ln^2\left(\frac{K_O}{F_O}\right),$$

$$\text{(6)}\quad \varepsilon_3 = 3\ \ln^2\left(\frac{K_O}{F_O}\right)\ \left(\frac{1}{3}\ln\left(\frac{K_O}{F_O}\right) - 1 + \frac{F_O}{K_O}\right)$$

For this calculation, we need the interest rate factor, the relevant option prices, the at-the-money strike K_0, the forward price F_0 and the strike intervals ΔK. All but the strike intervals have already been determined. At the smallest and largest strike, the strike interval is specified as the difference between that strike and the next. At all other strikes the strike interval is specified as half the distance between adjacent strikes.

As seen in Table 1, on July 28, 2010, the strike intervals at the extreme strikes as well as at intermediate strikes are all equal to 5 for S&P 500 August 2010 options selected for the calculation of SKEW.

With strikes intervals determined, all that remains to be done is :

- Calculate each of the components of the sums in P_1, P_2, P_3.
- Add the components up, multiply by the interest rate factor and add the adjustment term, the epsilons.

- Use the formula for S to calculate the price of skewness from P_1, P_2, P_3 at each expiration
- Interpolate or extrapolate the 30-day value of S.
- Calculate SKEW as 100 – 10S

The calculated values of the components of the three sums are shown in Table 2. The column labeled "for P1" contains the value $x = \frac{\Delta K}{K^2} * OptionMidquote$ at each strike, and it picks up the put midquote for strikes below the money, the call midquote for strikes above the money, and the average midquote at -the- money.

The column labeled "for P2" contains the value y $= 2x * (1 - \ln(\frac{K}{F_0})$ at each strike.

The column labeled "for P3" contains the value z = $3x * (2\ln(\frac{K}{F_0}) - \ln^2(\frac{K}{F_0}))$ at each strike.

Table 2. Sample of calculated values of components of SKEW

Strike	Put Midquote	delta k	Call Midquote	midcall - midput	for P1	for P2	for P3	Exposure to -10 * skewness portfolio
690	**0**		**416.45**					
695	**0**		**412.05**					
700	**0.075**	5	**407.8**	407.725	7.6531E-07	2.23E-06	-2.59E-06	0.009930169
705	**0.075**	5	**403.65**	403.575	7.5449E-07	2.19E-06	-2.5E-06	0.009681804
710	**0.075**	5	**398.3**	398.225	7.4390E-07	2.15E-06	-2.42E-06	0.009438799
715	**0.075**	5	**391.65**	391.575	7.3353E-07	2.11E-06	-2.34E-06	0.00920102
720	**0.075**	5	**387.8**	387.725	7.2338E-07	2.07E-06	-2.27E-06	0.008968339
...	...	...	...	...				
1095	**17.95**	5	**30.05**	12.1	7.4852E-05	0.000151	-4.86E-06	0.000103684
1100	**19.75**	5	**26.65**	6.9	8.1612E-05	0.000164	-3.05E-06	5.1038E-05
1105	**21.85**	5	**23.7**	1.85	9.3262E-05	0.000187	-9.37E-07	-6.63095E-07
1110	**24.25**	5	**21.35**	-2.9	8.6641E-05	0.000173	1.48E-06	-5.14376E-05
1115	**26.55**	5	**19.65**	-6.9	7.9028E-05	0.000157	3.47E-06	-0.000101303
1120	**29.2**	5	**16.05**	-13.15	6.3975E-05	0.000126	4.51E-06	-0.000150277
1125	**31.9**	5	**14.3**	-17.6	5.6494E-05	0.000111	5.47E-06	-0.000198376
1130	**35.05**	5	**11.95**	-23.1	4.6793E-05	9.16E-05	5.75E-06	-0.000245616
1135	**37.7**	5	**9.95**	-27.75	3.8619E-05	7.53E-05	5.75E-06	-0.000292014
...	...	...	...	...				
1205	**98.6**	5	**0.475**	-98.125	1.6356E-06	2.99E-06	7.98E-07	-0.000861405
1210	**102.15**	5	**0**	-102.15	0.0000E+00	0	0	0
1215	**108.5**	5	**0.525**	-107.98	1.7782E-06	3.22E-06	9.48E-07	-0.000931744
1220	**113.5**	5	**0**	-113.5	0.0000E+00	0	0	0
1225	**118.4**	5	**0.2**	-118.2	6.6639E-07	1.2E-06	3.85E-07	-0.000999623
1230	**123.55**	5	**0.175**	-123.38	5.7836E-07	1.03E-06	3.47E-07	-0.001032668
1235	**127.3**	5	**0.15**	-127.15	4.9173E-07	8.76E-07	3.06E-07	-0.001065131
1240	**132.7**	5	**0.375**	-132.33	1.2194E-06	2.16E-06	7.84E-07	-0.001097021
1245	**138.7**	5	**0.125**	-138.58	4.0322E-07	7.12E-07	2.68E-07	-0.001128349
1250	**142.7**	5	**0.075**	-142.63	2.4000E-07	4.22E-07	1.64E-07	-0.001159126
1255	**148.35**	5	**0.175**	-148.18	5.5555E-07	9.72E-07	3.92E-07	-0.00118936
1260	**153.35**	5	**0**					
1265	**158.35**	5	**0**					

Source: CBOE

Table 3. Final Results from August and September 2010 S&P 500 options

Trade Date	07/28/10	**P1 = E[R]**	-0.00173	**Trade Date**	07/28/10	**P1 = E[R]**	-0.0041
Expiration Date	08/20/10	**P2=E[R^2]**	0.003606	**Expiration Date**	09/17/10	**P2=E[R^2]**	0.00864
Time to Expiration= tau	0.065	**P3=E[R^3]**	-0.00049	**Time to Expiration**	0.142	**P3=E[R^3]**	-0.001
Treasury Bill Rate	0.00155	**Std. Dev. [R]**	0.060021	**Treasury Bill Rate**	0.00155	**Std. Dev. [R]**	9.29%
Forward Price	1106.85	**Skewness**	-2.19656	**Forward Price**	1106.45	**Skewness**	-1.68
Center Strike	1105	**SKEW @ 23 days**	121.9656	**Center Strike**	1105	**SKEW @ 51day**	116.75
Epsilon 1	1.40E-06	**VIX @ 23 days**	22.98	**Epsilon 1**	8.61E-07	**VIX @ 51 days**	24.01
Epsilon 2	-4.2E-06	**Delta Hedge Position**	-0.41	**Epsilon 2**	-2.58E-06	**Delta Hedge Position**	-0.62984
Epsilon 3	1.176E-11	**TBill Position**	100.00	**Epsilon 3**	4.442E-12	**TBill Position**	100.00

Source: CBOE

Table 3 shows the calculated values of the epsilons using equations (4) to (6) and based on forward prices and at-the-money strikes for the August and September 2010 expirations, and the final results from summing up the elements of P1, P2, P3 in each of their corresponding columns.

$P_1 = -e^{rT} \text{ sum } x + \varepsilon_1 \quad P_2 = e^{rT} \text{ sum } y + \varepsilon_2 \quad P_3 = e^{rT} \text{ sum } z + \varepsilon_3$

The three P values are now substituted in the formula for S to get the value of skewness at each expiration. The SKEW- like value for each expiration is shown in the table.

Note that the value of VIX is also shown in the table. VIX is easily derived from the elements in column 1. Specifically VIX = 100 * sqrt{-(T^{-1}) * P_1}

Oops.. .we almost forgot

<u>Last Step of Calculation</u> – *Calculate the 30-day weighted average of S_1 and S_2. Then take the difference between 100 and 10 times that weighted average to get SKEW.*

$$SKEW = 100 - S = 100 - 10 * \{w\ S_1 + (1-w)_2\, S_2\}$$

$$w = \frac{\text{minutes to next expiration} - \text{minutes to 30 days}}{\text{minutes btw next \& near term expirations}}$$

When the near-term options have less than 30 days to expiration and the next-term options have more than 30 days to expiration, the resulting SKEW value reflects an interpolation of S_1 and S_2 ; i.e., each individual weight is less than or equal to 1 and the sum of the weights equals 1.

At the time of the SKEW "roll," both the near-term and next-term options have more than 30 days to expiration. The same formula is used to calculate the 30-day weighted average, but the result is an extrapolation of S_1 and S_2 ; i.e., the sum of the weights is still 1, but the near-term weight is greater than 1 and the next-term weight is negative (e.g., 1.25 and –0.25).

Continuing with the July 28, 2010 example,

S = 0.730208333 * (-2.19656) + 0.269791667 *(-1.68) = -2.056

SKEW = 100 – 10*(-2.056) = 120.56

Special Note: All CBOE Volatility Indexes – SKEW,VIX, VXD, VXN, RVX, VXV, OVX, GVZ and EVZ – are calculated using option price quotes from CBOE exclusively.

References

D.Backus, K. Li, S. Foresi and L.Wu, “Accounting for Biases in Black Scholes,” Fordham University Working Paper 30, 1997.

Bakshi, G., Kapadia, N. and Madan, D., “Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options,”, *The Review of Financial Studies* 16(1):2003, 101–143.